B.Sc. Part-I (Sub) Set theory 11-07-2020

real and algebraic number and System

Q.1 The set R of all real number is uncountable.

Proof. Since the set R of real numbers and the set of unit interval [0,1] are equivalent sets. Hence we have to prove that the set R is uncountable it is sufficient to prove that the set [0,1] is uncountable. Let [0,1] is a countable set. Then we can write it as an infinite sequence $a_1, a_2, a_3, \ldots a_n, \ldots$ where every number in [0,1] occurs among the a_r. $r \in N$.

We know that a subset of a countable set is countable set. Hence, if the set R is countable then set [0,1] must be countable. But we know that ~~a subset set~~ [0,1] is uncountable. Therefore the set R of all real number is uncountable.

Proved.

Q.2. Show that the set of all irrational numbers is uncountable

Soln: Since the set R of all real numbers (that is, the set of all rationals and irrationals) is uncountable. But we know that the set R of all real numbers is uncountable. and the set Q of rational numbers is countable. But we know that the set of all positive rationals is countable. Then from ~~the~~ B a countable subset of an uncountable set A, then A, then A-B is uncountable. It follows that the complement of the set of rationals relative to the set of real numbers, that is R-Q of irrational numbers is uncountable.

Q.3. Prove that the set A of algebraic numbers is countable

Soln: Let $P = \cup \{P_n(x) : P_n(x) = 0, n \in N\}$, where $P_n(x)$ stands for a polynomial of degree n with integral coefficients, is enumerable. Since $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_nx^n$ with integral coefficients is enumerable.

Since, an algebraic number is the root of a polynomial $P(x) = 0$ with integral coefficients.

Let $A_n = \{x : x \text{ is a solution of } P_n(x) = 0\}$

Since, a polynomial of degree n has at most n roots and hence A_n is finite.

Now, $A = \cup \{A_n : n \in N\}$, is the set of algebraic numbers and it is union of countable sets and hence

Rest part of Q.3.

A, the set of all algebraic numbers is uncountable. Again, A is not finite hence A is enumerable.

Q.4. Prove that the set of all transcendental numbers is uncountable.

Soln: Since, Transcendental numbers are that real numbers which are not algebraic numbers.

Let T be the set of all transcendental number and A be the set of all algebraic numbers, then $T \cup A = \mathbb{R}$, the set of all real numbers.

Let T be countable and we know that A be countable then $T \cup A$, i.e. $\mathbb{R}$ be ~~then~~ countable. But the set $\mathbb{R}$ of real numbers is not countable. Thus we have a contradiction.

Hence, the set T of all transcendental numbers is uncountable.

Q.5. Show that if B is a countable subset of an uncountable set A, then $A-B$ is uncountable.

Soln: Let $A-B$ is countable. Then $(A-B) \cup B$, would be countable, since it is union of two countable sets. We know that the union of a countable family of countable sets is countable.

Again, $\because B \subseteq A$

$\therefore$ We have $(A-B) \cup B = A$.

Thus, A would be countable, which contradicts the hypothesis that A is uncountable set.

Hence, we conclude that $A-B$ is uncountable.

Proved